Drawing and use of auxiliary projection nets (the program STEGRAPH)

A. F. MENDES
J. C. KULLBERG

Centro de Estratigrafia e Paleobiologia da UNL, Faculdade de Ciências e Tecnologia,
Quinta da Torre, P-2825 Monte de Caparica, Portugal

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RESUMO

Palavras-chave: Projecções — Redes — Cristalografia — Geologia Estrutural — BASIC — MS-DOS.

Apresenta-se o desenvolvimento de um algoritmo para a construção das redes auxiliares de projecção (conforme, equivalente e ortográfica) nas suas versões equatorial e polar. Apresenta-se também o algoritmo para o traçado da rede de contagem de pontos IGAREA 220 (ALVES & MENDES, 1972). Estes algoritmos são a base do programa *STEGRAPH* (ver. 2.0), para computadores MS-DOS que, além do desenho de redes, permite outras aplicações.

RÉSUMÉ

Mots-clés: Projections — Canevas— Crystallographie — Géologie Structurale — BASIC — MS-DOS.

On presente le dévelopement d'un algorithme pour la construction de canevas auxiliaires de projection (conforme, équivalente et ortographique), dans les vérsions équatoriale et polaire. On présente aussi l'algorithme pour le déssin du canevas de contage de points IGAREA 220 (ALVES & MENDES, 1972). Ces algorithmes sont la base du programme *STEGRAPH* (ver. 2.0) qui permet des applications autres que le dessin des canevas.

ABSTRACT

Key-words: Projections — Nets — Crystallography — Structural Geology — BASIC — MS-DOS.

The development of an algorithm for the construction of auxiliary projection nets (conform, equivalent and orthographic), in the equatorial and polar versions, is presented. The algorithm for the drawing of the "IGAREA 220" counting net (ALVES & MENDES, 1972), is also presented. Those algorithms are the base of *STEGRAPH* program (vers. 2.0), for MS-DOS computers, which has other applications.

INTRODUCTION

Since the time when man faced the reality of Earth being, in a first approach, a spherical body, he had the problem of, for a correct representation of its surface characteristics, develop methods allowing to planify a spherical surface sacrificing the least possible rigor in the representation.

Several methods for the projection of the characteristics of a spherical surface on a plane were then developed, within which we can mention the cylindrical projections, including the Mercator projection, widely used in navigation charts, the conical projection, as the Gauss projection, the polar projections, and so on.

Within such projections some were, by its ease and characteristics, of a widespread use in the Earth Sciences, mostly three of them: the gnomonic projection, not further mentioned in this work, and the stereographic and Lambert azimuthal projections, to which we can add the orthographic projection with applications, within others, to studies of deformation at all scales.

The graphic bidimentional representation of tridimentional units with geological interest and significance, reducible to lines and planes as, for example, crystal faces, crystallographic axis, stratification and schistosity planes, fractures, fold axis, fault grooves, etc., is universally made through the so called *Stereographic Projection*, which so became an indispensable "tool" in the daily work.

The recent development of powerful, widespread and user-friendly computer systems have been arising a growing interest in the development of programs in this area, turning the classical manual methods, time consuming and not precise enough, obsolete, namely in scientific research involving a great number of data.

The use of stereographic projection nets is also, in the teaching of the geological sciences, a permanent concern, namely in subjects including Crystallography, Mineralogy, Structural Geology, Geological Mapping, Engineering Geology and others. We deem more correct, from a strictly pedagogic point of view, and taking in account an easier understanding of the principles used, that the students start those matters using the refered manual methods.

To help calculations and drawings in each of those projections, nets of several diameters (normally 10, 15 or 20cm) can be purchased, the most known being the Wulff net (equatorial stereographic projection) and the Schmidt net (equatorial Lambert projection). The users must have good quality auxiliary projection nets (conform or equal area, depending on the work), meaning graphically correct and non deformed. However, those nets are not easy to obtain and, when available, are not drawn with enough rigor for a acceptable representation allowing calculations with the desired precision. Thus, with the objective of surpassing that difficulty, we decided to construe a computer program for the precise drawing of auxiliary projection nets (presented in annex).

Nevertheless, while the program was growing, we decided to widen its scope from a strictly pedagogic character to areas of a general use, namely as an auxiliary instrument in scientific work, either fundamental or applied. It is also in this area that great gaps in the commercially available programs, at least in those most accessible, show themselves.

The main objective of this work is to present a computer base for the drawing of equiarea equivalent and orthographic auxiliary projection nets, in its equatorial and polar versions, as well as the *IGAREA 220* counting net (ALVES & MENDES, 1972), presenting the base algorithm for the program *STEGRAPH* (now in the Vers. 1.4), written in Microsoft™ QuickBasic® 4.5 (for computers using MS-DOS or DR DOS systems).

This is the preliminary phase of a work with the objective of developing a stereographic projection program allowing the interactive treatment and analysis of data, namely in the areas of Structural Geology and Crystallography.

I. THEORETHICAL PRINCIPLES

I.1. Projection fundamentals

I.1.1. Stereographic Projection

The *stereographic projection* can be defined as the projection of a spherical surface (Miller sphere) on a plane (Fig. 1), where the *projectant rays* are centered in a *point of view* placed in the upper (*zenith*) or lower (*nadir*) poles of the sphere of reference (it is not, thus, a orthographic or a perspective projection), being the projection plane coincident with its *equatorial plane*. The great circle corresponding to the intersection of the Miller sphere with the equatorial plane and its limiting circumference are called, respectively, *fundamental circle* and *primitive*.

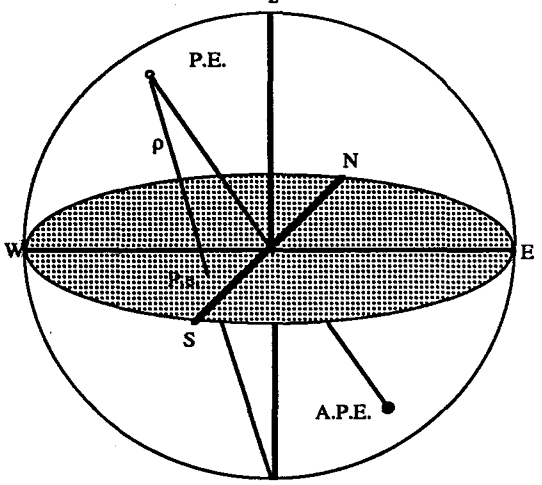


Fig. 1 — Stereographic Projection (perspective of the Miller sphere). z - zenith; n - nadir; P.E. - spheric pole; A.P.E - spheric antipole; P.e. - stereographic pole; ρ - projectant ray.

We can thus represent any point of the spherical surface but the point of view itself in the equatorial plane, connecting the *spheric pole* to the point of view with a *projecting ray*, and defining the so obtained *stereographic pole* as the intersection of the projection ray with the equatorial plane. So, and reporting to the same figure:

— let's consider the sphere of reference split in two hemispheres (*upper* and *lower hemispheres*) separated by an horizontal equatorial plane;

— let's consider, passing by the center of the sphere and at right angles to the equatorial plane, a line, the *zenith-nadir line*. That line will intersect the sphere in two points, the *upper* and *lower points of view*.

If we cross the sphere by any other line, passing by the center, two antipodal intersections will result, its *spheric pole* and *antipole*;

— if, on the other hand, we cut the sphere by a plane that also includes its center, an intersection corresponding to a *great circle* will result, or its *cyclographic* representation.

Let's adopt the lower point of view, as normally is the case with this projection (Fig. 2); that way only the part of the geometric elements located on the upper hemisphere will be of interest to us, otherwise the projections would fall outside the primitive.

If we connect any spheric pole (the cyclographic representation of a plane is also made of points) with the point of view, a new line will result, crossing the equatorial plane somewhere in a point corresponding to the *stereographic pole*, or its stereographic projection.

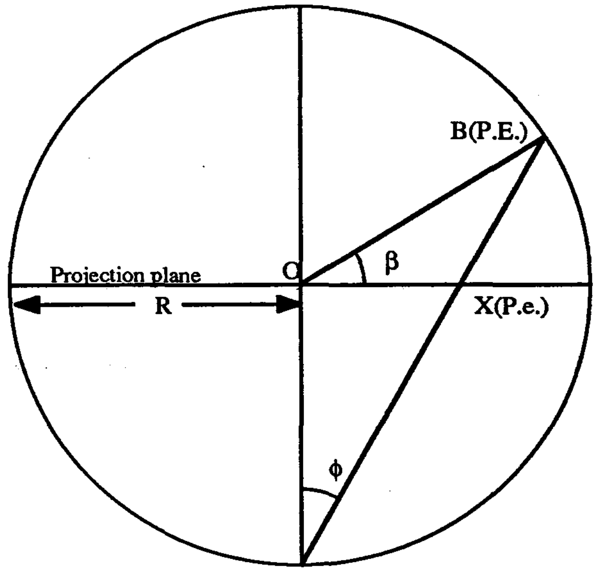


Fig. 2 — Stereographic Projection (profile of the Miller sphere); O - center of the sphere and projection; R - radius; n - point of view; B(P.E.) - spheric pole; X(P.e.) - stereographic pole; β - dip of a plane; ϕ - angle inscribed in the complementary of β.

I.1.2. Lambert projection

In the *Lambert* (or *equal area*) *projection* (Fig. 3), as the lower hemisphere is normally used, the projection plane is tangent to the nadir of the sphere, and the representation of the spheric poles is not obtained by projectant rays but by drawing an arc from the spheric pole to the projection plane, centered in the point of tangence and with radius equal to the distance from that point to the spheric pole. We can thus define the *Lambert pole* as the intersection of the arc so defined with the plane of projection.

I.1.3. Orthographic projection

In the *orthographic projection* (Fig. 4), using also, hypothetically, the lower hemisphere, the plane of projection is also tangent to the nadir of the sphere. The orthographic representation of the spheric poles (the *orthographic poles* — P.O.) is obtained by the intersection of the plane of projection by lines at right angles to it passing by the spheric poles.

By those processes one dimension of space can thus be eliminated from any of the geometrical elements of interest to us (the planes are now represented by lines — great circles — and the lines by points). Nevertheless, they are difficult and slow to calculate, for which auxiliary projection nets are normally used, as refered.

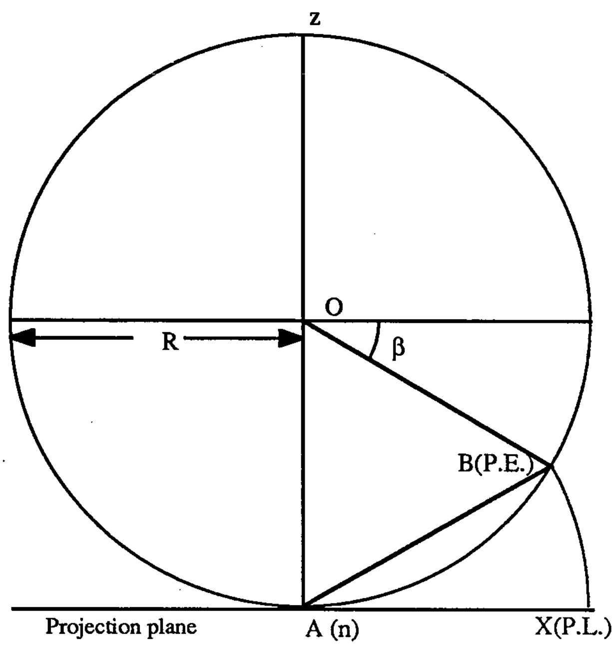


Fig. 3 — Lambert projection (profile). O - center of the sphere; A - tangency point (center of the projection); R - radius of the sphere (different from the radius of the projection); B(P.E.) - spheric pole; X(P.L.) - Lambert pole; β - dip of plane.

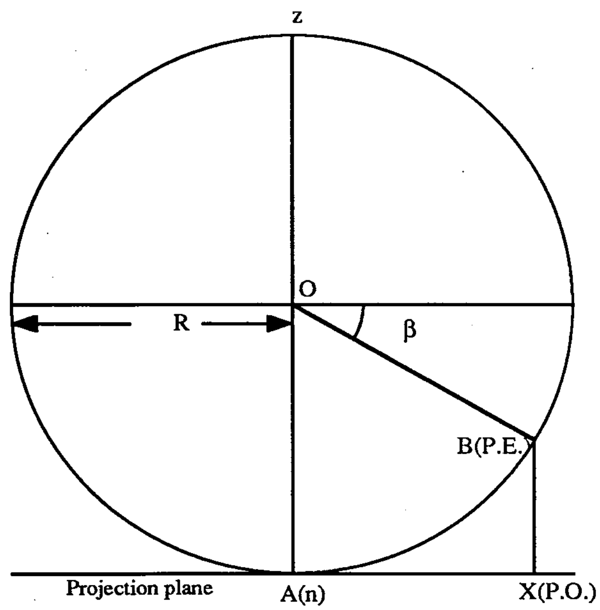


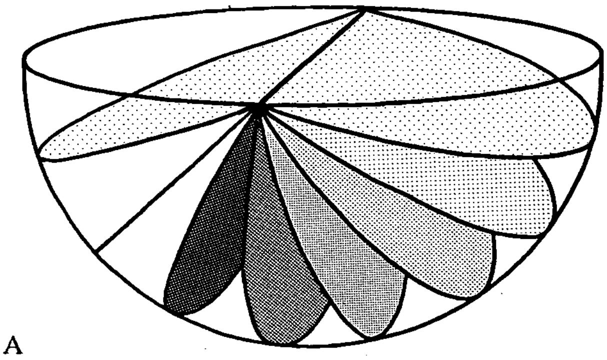
Fig.4 — Orthographic projection (profile); O - center of the sphere; A - Point of tangency (center of the projection); R - radius of the sphere (equal to the radius of the projection); B(P.E.) - spheric pole; X(P.O.) - orthographic pole; β - dip of plane.

I.2. Construction of auxiliary projection nets

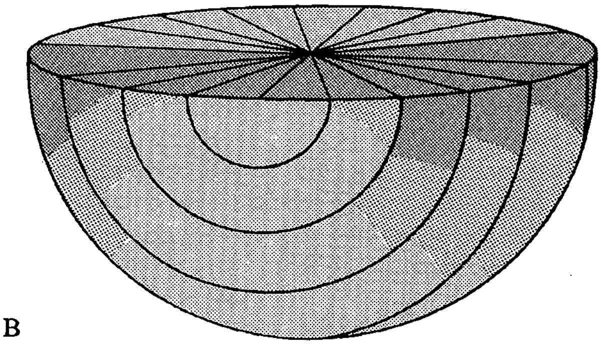
I.2.1. True-angle stereographic net (Wulff net)

I.2.1.1. Equatorial net

This net is composed by two families of curved lines (great and small circles), drawn from the projection on the equatorial plane of the Miller sphere of two such families of planes (Fig. 5C).



A



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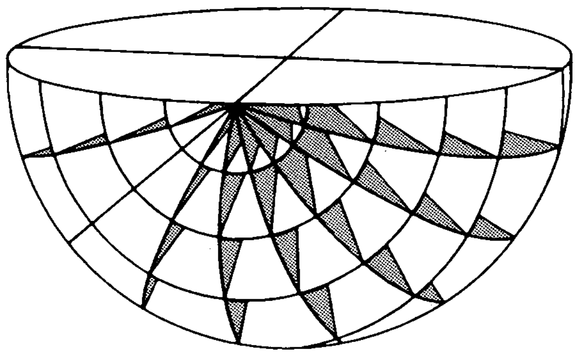


C

Fig. 5 — (A) - Family of great circles, intersecting in a common diameter of the primitive; its stereographic projection defines the meridians of the Wulff net. (B) - Family of apex centered coaxial cones, whose stereographic projection defines the family of small circles in the Wulff net. (C) - Conjugation of the two families defined in (A) e (B).

The arcs of the great circles correspond to the projection of a family of planes with dips differing from a certain angular value, all of them passing by a common line on the equatorial plane. This line is the N-S line of the projection (Fig. 5A).

The arcs for the small circles correspond to the projection of a series of planes cutting the sphere of reference at right angles both to the plane of projection and the aforementioned N-S line. Those planes are so separated as to the centered angles, in the equatorial plane, between the successive radii so defined, differ by the stipulated amount defined for the construction of the net. The plane passing by the center of the sphere of reference will be the only great circle of the family, and its projection will be the E-W line of the net.

The surfaces defined by the connection of the center of the sphere with the intersections of the successive planes with the sphere are cones of revolution, coaxial and with juxtaposed vertices (Fig. 5B).

In this projection, the shapes are maintained, and all the projected planes are represented by arcs of circumference and the angles are projected in true value. This fact allows us to establish a series of relationships between the great circle of any plane and its stereographic projection on the net.

Let's consider the vertical section of the sphere of reference (Fig.2), perpendicular to any given plane as to represent its true dip (β); the segment OB (the trace of the plane on the section plane) is the radius of the sphere, and the segment OX corresponds to the distance between the great circle of the given plane and the center of the sphere, measured on the fundamental circle, of radius R. The line nB is the projectant line from B (thus connected to the point of view n) passing, necessarily, by X.

So, as the angle OT^TB (ϕ) is the inscribed angle corresponding to the complementary of the b centered angle, its value is:

$$\phi = (\pi/2 - \beta)/2 \quad \Rightarrow \quad \phi = (\pi/4 - \beta/2) \qquad [1]$$

from which:

$$OX = R \,.\, tg(\pi/4 - \beta/2) \qquad [2]$$

I.2.1.2. Polar net

This net is also composed of two families of lines, the first straight and radial and the second circular and concentric.

The principle of construction for this net is identical to the previous one, differing only on the following aspects:

1- The family of planes originating the great circles have its common line not on but at right angles to the equatorial plane, passing by its center;

2- The succession of parallel planes originating the small circles is oriented not perpendicularly to the primitive but parallel to it.

So, the difference between the polar and equatorial nets consists in a 90° rotation of the planes necessary to their construction, on the first case on the N-S line of the projection and on the last on the zenith-nadir line of the sphere of reference; the same is valid for the nets in the other projections.

I.2.2. Equal Area (Schmidt) net

This net is built similarly to the equatorial Wulff net, although its projection is obtained not by geometrical means, but by calculation, as shown in figure 3.

A vertical section on the reference sphere shows its intersection (B) with a plane of dip β passing by the center O. The projection of B is made by rotating it to X, on the projection tangent plane, maintaining AX = AB. This distance, AX, can be calculated from the radius of the sphere, R as follows:

$$AX = 2\,R \sin(\pi/4 - \beta/2) \qquad [3]$$

This expression causes the radius of the primitive to be greater than the radius of the sphere, as can be proved making b equal to 0° (the very plane of the primitive). So, to make $AX_{max} = R$, all AX values have to be scaled.

As, for $\beta = 0°$, $\quad AX = R\sqrt{2}$,

so:

$$AX = \sqrt{2}\,R\,sen(\pi/4 - \beta/2) \qquad [4]$$

for any plane of dip β.

I.2.2.2. Polar Net

This net is constructed the same way of the polar equiangular net, differing only in the way its concentric circles are scaled (eq. [4]).

The polar nets are presented because, while of more restricted usage, they ease the projection of points. On those nets the rotation of the transparent paper is unnecessary, substantially speeding the projection of a great quantity of point data. As this is their main use, it is natural that only the polar equal area net is used, as it is the only one adequate to a later statistical treatment on counting nets.

I.2.3. Orthographic Projection

I.2.3.1. Equatorial Net

A vertical section of the reference sphere shows a plane of dip β intersecting it in B and passing by its center O. The point X is found projecting B orthogonally to the projection plane, being the distance AX found from the radius R as:

$$AX = R \cos(\beta) \qquad [5]$$

I.2.3.2. Polar Net

The construction of this net obeys the principles of construction of the other two polar nets, the only difference being that the radius of its concentric circles are found through the equation [5].

I.3. Statistical treatment of point populations

I.3.1. Counting nets

For the statistical analysis of a three dimensional population of orientations, such as planes or lines, its projection is necessary on one of the nets described above. Nevertheless, as only the Lambert projection respects the equality of areas and, consequently, the density of the distribution on the whole surface, it is natural that this is the projection of choice for this type of analysis.

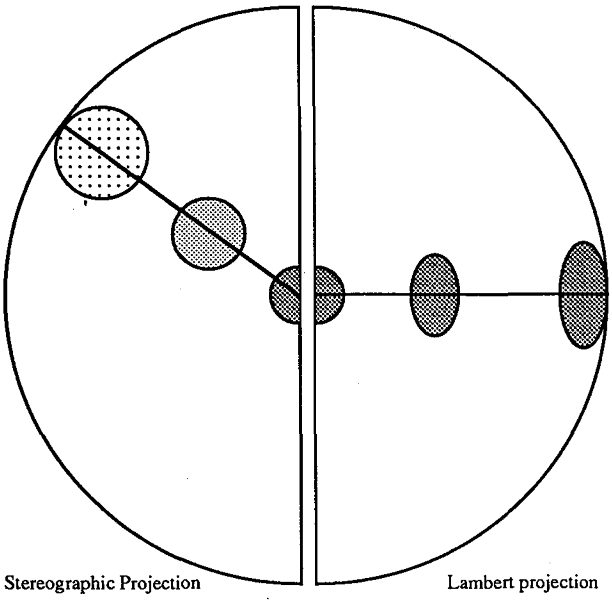


Fig. 6 — Comparison between the deformations induced by the stereographic and the Lambert projections.

As we can see in the figure 6, the stereographic projection, while keeping the shapes of equal aperture circles undistorted, deforms their area, nearly doubling it from the center to the primitive. On the opposite, the Lambert projection (to which the Schmidt net is associated) respects the equality of areas, even if it has to sacrifice the shape to do so; the spheric circles of equal aperture then change, when projected, to more and more deformed ellipses from the center to the primitive.

Several types of counting nets exist, described by a number of authors, every one of them based on the same principle, the creation of a regular net of counting centers, each one of them taking the value ("weight") of the number of points found in a certain "area of influence". From the several nets we can refer those of hexagonal symmetry and polygonal area of influence, used by several authors (for instance, the Kalsbeek net), those of quadratic symmetry and elliptical area, not very much used now, and sundry nets of several symmetries with circular areas of influence. We chose this type as those nets represent, in our understanding, the ones allowing a more correct statistical analysis, attributing to each counting center areas of influence represented by small circles of constant radius.

ALVES & MENDES (1972) developed two counting nets (IGAREA 220 and IGAREA 523, numbered from the counting cones used), the first for distributions between one hundred and two hundred points and the second for distributions of a high number of points or smaller distributions of weak concentration. The second net is, from personal experience, seldom used, not only because the number of points lies normally between one hundred and two hundred but also the drawing is so dense that its use becomes very awkward. For that reason the authors separated it in four components, to be used separately and then superposed.

Those nets were calculated admitting a constant angular distance between counting centers, 10° in the case of IGAREA 220 (in annex), the only to be drawn with the program STEGRAPH. We have, then, the counting centers distributed by concentric circles with radii growing by 10° steps from the center, and also separated within each circle by approximately 10°. In practice, to maintain an hexagonal symmetry, chosen by those authors, the number of centers in each "shell" was rounded to the nearest multiple of 6. The radius of each "area of influence" is 8°, allowing a sufficient superposition.

II. DRAWING OF NETS

II.1. Equatorial nets

The calculation and drawing of equatorial nets are somewhat difficult due to the fact that both the drawing of great and small circles are made for every point.

Let's consider, then, a great circle corresponding to a plane of dip β (Fig. 7). Whatever its value, it is equal to the plunge of the line of maximum plunge in the plane, represented by a point in the intersection of the cyclographic representation of the plane and the diameter of the primitive perpendicular to the horizontal line of the plane. We have, then, three of its points, corresponding respectively to the line of true dip and the two points of intersection of the cyclographic representation with the primitive.

To calculate the position of the other points of the plane, we must first find its apparent dip β' in all directions between the horizontal and the line of true dip (Fig. 8)

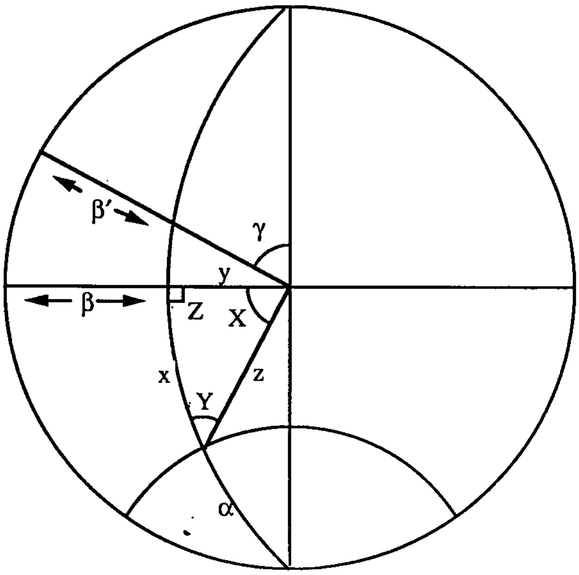


Fig. 7 — Calculations for arcs in equatorial nets.

As we can see from the figure,

$$\tan(\beta) = \frac{b}{a} \quad \text{and} \quad \tan(\beta') = \frac{b}{c} \; ;$$

so then:

$$b = a \, . \tan(\beta) \quad \text{e}$$
$$b = c \, . \tan(\beta') \quad ,$$

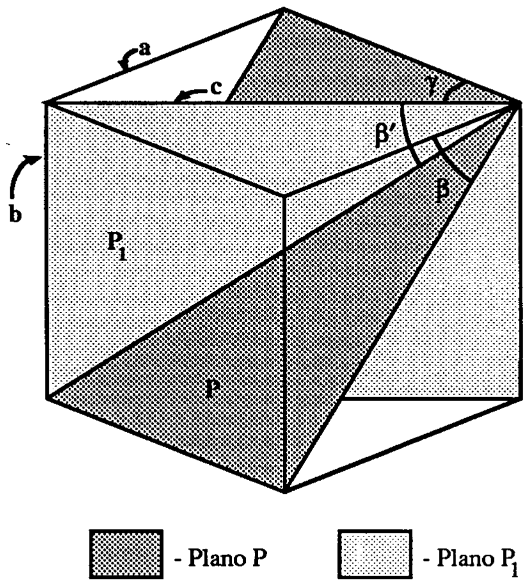


Fig. 8 — β - True dip of plane P; β' - dip of plane P in plane P_1; γ - Angle between the strikes of the plane P and the sectioning plane P_1.

which follows as:

$$a \, . \tan(\beta) = c \, . \tan(\beta').$$

From that we take:

$$\tan(\beta') = \frac{a}{c} \, . \tan(\beta) \quad ;$$

but as:

$$\frac{a}{c} = \sin(\gamma) \quad ,$$

we have:

$$\tan(\beta') = \sin(\gamma) \, . \tan(\beta) \quad ,$$

or finally:

$$\beta' = \tan^{-1}[\sin(\gamma) \, . \tan(\beta)]. \qquad [6]$$

Then we only must perform the necessary calculations, according to equations [2], [4] and [5] (Wulff, Schmidt and orthographic nets, respectively); the calculations must be made for a large enough number of points along the line so that it can be drawn with a minimum of precision. In the *STEGRAPH* program, for reasons of clarity, the circles of dip multiple from 10°, besides being drawn with a different colour from the others, are also drawn between the small circles of radius equal to the equidistance of the circles; those not multiple of 10° are drawn, in the Wulff net, between the small circles of radius 10° and, in the Schmidt and orthographic nets, between the small circles of radius 20°.

The problems arising from the drawing of small circles are different from those seen in the drawing of great circles, slightly more difficult but nonetheless solvable with the help of the spherical trigonometry.

Let us consider, then, a small circle of radius a (Fig.7). This circle defines, in the family of great circles whose axis intersects the small circle in its middle arcs of value $x = 90° - \alpha$. Admitting that the dips of such great circles are successively known, for instance fixed, by the program, as equal to β, we can then define a spherical triangle, rectangular in Z, whose sides are x and y and the hypotenuse is z. As in any rectangular spherical triangle the cosinus of the hypotenuse is equal to the product of the cosinus of the sides, we have:

$$z = \cos^{-1} (\cos x \, . \cos y)$$

But, in the polar coordinate system used up to this point (where the position of each point is defined by two coordinates, an angle θ and a distance L) z, besides being the hypotenuse of the triangle considered is also the value of the arc corresponding to the distance L, which can be calculated from the equations [2], [4] and [5].

As we can also see, the angle θ is equal to the angle X of the spherical triangle used and, to find its value, we must use another property of the spherical triangles, which states that in any spherical triangle the sinuses of the sides are proportional to the sinuses of the angles they oppose. We have, then:

$$\frac{\sin x}{\sin X} = \frac{\sin z}{\sin Z}$$

so,

$$Z = \arcsin\left(\frac{\sin z \,.\, \sin X}{\sin x}\right) \qquad [7]$$

Varying, with a convenient step, the dip β of the great circle, and taking as constant the angle α, we can calculate the polar coordinates of all the points belonging to the small circle and, with a simple transformation, the orthogonal coordinates used by the drafting plotter.

II.2. Polar Nets

Allowing that, obviously, the calculations for the drawing of both the equatorial circle and the great circles (the "meridians" of the Miller sphere) poses no problems (today any drafting plotter has an internal set of instructions allowing the drawing of both straight lines beginning and ending at any point within the drawing area and circles of any radius centered at any point), the drawing of a small circle is dependent on the ratio between its radius on the sphere (in degrees) and its radius on the projection (or the ratio between its radius and that of the primitive).

We thus opted for drawing the net in two steps, the first common to every type of polar net consisting on the drawing of the circle of the primitive, the angular values and the two main great circles (the horizontal and the vertical ones). From this point, by the equations [2], [4] and [5], we calculate the radius of the successive small circles, drawing them centered within the circle of the primitive. Note that, if desired, the primitive and the small circles of radius multiple of 10° can have a different colour, for clarity.

After the small circles, the drawing of the great circles is simple, in this type of net they are represented by straight lines radiating from the center. For clarity, we do not draw them from the center. The multiples of 10° are drawn from the first small circle, and the other are drawn from the 10° small circle.

II.3. The IGAREA 220 counting net

For the counting nets, in the present case the IGAREA 220 net, already described, the process is based on the drawing of small circles of fixed radius (8° in this case) centered on the knots of the net. As the projection employed is the Lambert equal area projection, the small circles are projected as closed 4^{th} degree curves (Fig. 9).

So, the drawing of a small circle depends from a number of factors, among them the coordinates of its center. After they are calculated, thus knowing the position of the circle within the stereogram, the next step involves the calculus of the coordinates of all the points at a given angular distance from the center.

Given, thus, a small circle of coordinates **a** (distance to the center, in degrees) and **D** (angle

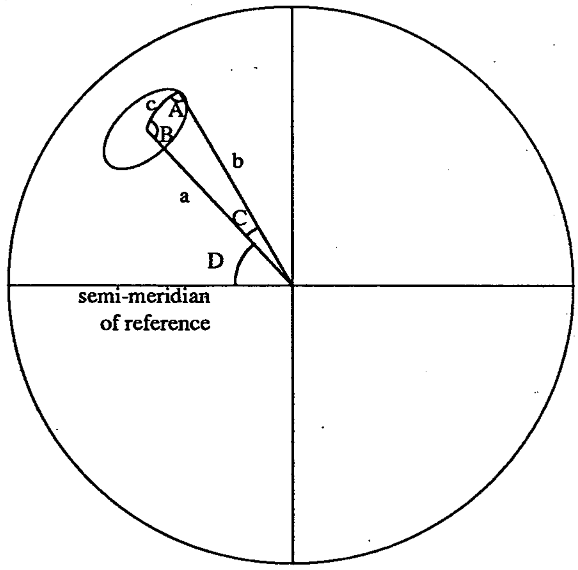


Fig. 9 — Drawing of small circles by calculating the coordinates of all its points.

between the vector radius of its center and the meridian of reference), the polar coordinates (L and θ) of any point P, at a set distance from the center, can be calculated as follows:

— Admitting a spherical triangle with its vertices placed respectivelly on the center of the stereogram, the center of the small circle and the point P, we can define the side **a** and the angle **D** (coordinates of the center of the small circle), and the side **c** (arc between the point **P** and the center of the small circle);

— Varying then the angle **B** between 0° and 360°, and knowing that:

$$b = \cos^{-1}[\cos(a)\,.\,\cos(c) + \sin(a)\,.\,\sin(c)\,.\,\cos(B)],$$

we can thus calculate the side **b**, which is actually the coordinate L, in degrees, of the point **P**:

— by the ratio

$$\frac{\sin(A)}{\sin(a)} = \frac{\sin(B)}{\sin(b)} = \frac{\sin(C)}{\sin(c)}$$

we can see that we can calculate the angle C that, added to the angle D, gives us the coordinate θ of the point P. For ease of the calculations, we chose to split the drawing in two parts, one where the angle C is considered positive, varying B between 0° and 180°, and repeating the calculations for C as negative. The points where C is equal to 0° (B = 0° or 180°), corresponding to discontinuities of the curve, are calculated by itself (we cannot, with these points, define a spherical triangle). The circles centered on the primitive are drawn taking in consideration that they cannot extend beyond the circle of the primitive. As a comment, we can add that this method can be used for the representation of any circular counting net, with any distribution of the counting centers, needing only their coordinates and the radius of the circles of influence. In the present case, we chose not to calculate those coordinates but to have them preset within the program.

II.4. The Ortographic DTP Abacus

By personal suggestion of A. Ribeiro, an ortographic abacus for use in Structural Geology was developed, based on the methods devised by DE PAOR (1983, 1986), and included in the program *STEGRAPH.*

This abacus consists of a series of circles projected ortographically on the small circles of an equatorial ortographic net, in order to calculate the deformation occurring along the small circles.

III. APPLICATIONS (Prog. STEGRAPH, vers. 2.0)

The program STEGRAPH (3) allows us, besides the drawing of the projection nets, and based on the fact that all the principles subjacent to the several projections are already in it, its use in other applications (Fig. 10).

III.1. Counting of points in the IGAREA 220 net

For the automatic counting of points we need to introduce the orientations of the planes or lines to project in an external text file (ASCII format) then used by the program. The orientations must be placed in consecutive lines, with a space between the two

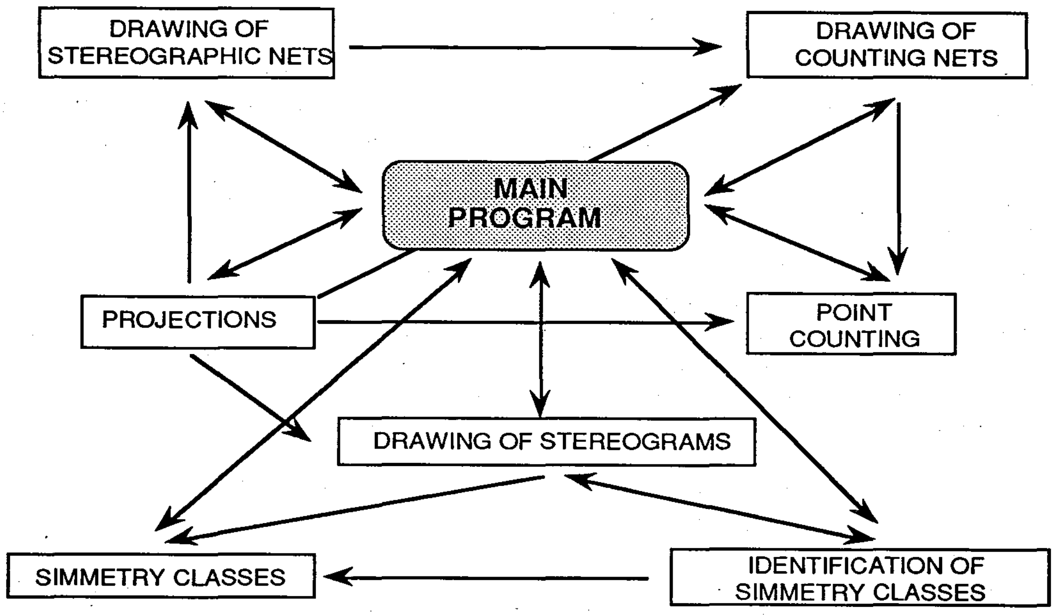


Fig. 10 — The program STEGRAPH.

values, and several formats can be used simultaneously:

— <u>Planes</u> (The examples refer to the same plane)
Normal notation: N45W 25SW or S45E 25SW
Azimuthal notation: 315 25SW or 135 25SW
Dip orientation: 225 25

— <u>Lines</u> (The examples refer to the same line)
Normal notation: 50 S40W
Azimuthal notation: 50 220

Internally all those formats are converted to azimuthal coordinates, easier to use by the program.

The counting is made (Fig. 11) calculating the angle (α) between the points included in the file (P) and the centers of the counting net (Q), adding to the value associated to each one of them when the angle is equal or less than 8°, by the equation:

$$\alpha = \cos^{-1} [\cos(\beta)\cos(\gamma) + \sin(\beta)\sin(\gamma)\cos(A)] \quad [8]$$

As the operation is processed reading successively every orientation in the text file in the hard disk, the number of points to use is only limited by the size of the hard disk.

III.2. Deduction of simmetry classes

As a complement to the applications strictly based on the stereographic projection and variants, and foreseeing its future use as a calculation basis for a new version of the program, we inserted a new option allowing the deduction of the simmetry class of a crystallographic model from a minimum number of its simmetry elements.

The existence of this subroutine is linked to a future projection of the stereograms of the various classes, associated with the stereographic projection of crystallographic models, to be included futurelly in the program.

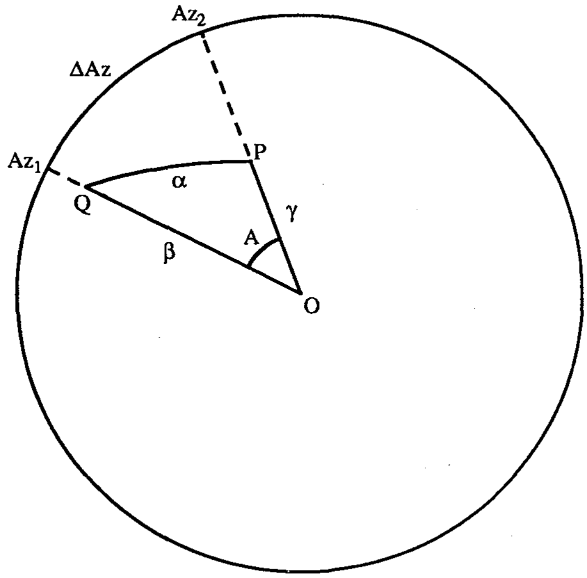


Fig. 11 — Calculus of the angle between two points (P and Q): A - difference between the azimuths of P and Q (= ΔAz); β and γ - Distance, in degrees, to the center (complementar to the respective dip values).

CONCLUSIONS

The present paper intended not only to demonstrate that the use and drawing of auxiliary projection nets ought not to be treated as a "hermetic science" but also to allow resolution of problems in the areas of crystallography and structural geology, namely:

1 — Applications in Crystallography:

a) Stereographic projection of crystallographic models;
b) Stereographic calculations (Miller indexes, for example);
c) Drawing of stereograms of all symmetry classes;

2 — Applications in Structural Geology:

a) Statistical analysis of populations of projected points;
b) Determination of stress fields from the analysis and interpretation of structures.

In its present version, the program STEGRAPH has an essentially pedagogic utilization; with the above mentioned additions, it shall be a tool extending to all the fields where the stereographical projection is used.

BIBLIOGRAPHY

ALVES, C.M. & MENDES, F.P. (1972) — Contagem manual de orientações. *Rev. Ciênc. Geológicas*, Lourenço Marques, Vol.5, Série A, pp. 9-17, 4 fig.

BEVIS, M. (1987) — Computing relative plate velocities: a primer. *Mathematical Geology*, New York, Vol.19, Nº6, pp.561-569, 2 fig., tabl.1.

BORGES, F.S. (1982) — Elementos de Cristalografia. 644 pp., *Fund. C. Gulbenkian*. Lisboa.

BRAVO, M.S. & MENESES, L.L. (1990) — Cristais a duas dimensões. 115 pp., *C.E.P.U.N.L.(INIC) - (FCT/UNL)*. Monte de Caparica.

DE PAOR, D. G. (1983) — Ortographic analysis of geological structures - I. Deformation theory. *Journal of Structural Geology*, Oxford, vol. 5, nº 3/4, pp. 255-277, 27 fig.

DE PAOR, D. G. (1986) — Ortographic analysis of geological structures - II. Practical applications. *Journal of Structural Geology*, Oxford, vol. 8, nº1, pp. 87-100, 19 fig.

HINKS, A.R. (1912) — Map projections. *Cambridge University Press*. Cambridge, 138 pp.

HOBBS, B.E., MEANS, W.D. & WILLIAMS, P.F. (1976) — An outline of Structural Geology. *John Wiley and Sons*, New York, 592 pp..

JAMSA, K. (1989) — Using DOS 4. *Osborne McGraw-Hill*. Barkeley, 1022 pp.

KULLBERG, M.C. & SILVA, J.B. (1981) — Apontamentos sobre o uso da projecção estereográfica em Geologia Estrutural. *A.E.F.C.L.* Lisboa, 34 pp., 52 fig., 2 tabl.

LIBAULT, A. (1975) — Geocartografia. Companhia Editora Nacional. São Paulo, 398 pp.

MORRISON, G.J. (1911) — Maps, their uses and construction. (2ª ed.). *Edward Stanford.*, London, 164 pp.

NAMEROFF, S. (1989) — QuickBASIC®, the complete reference. *Osborne McGraw-Hill.*, Barkeley, 593 pp.

PHILLIPS, F.C. (1960) — The use of stereographic projection in structural geology. (2ª ed.). *Edward Arnold (Publishers) Ltd.*, London, 94 pp.

PHILLIPS, F.C. (1978) — An introduction to Crystallography. (4ª ed.). *Oliver Boyd*, Edinburgh, 404 pp.

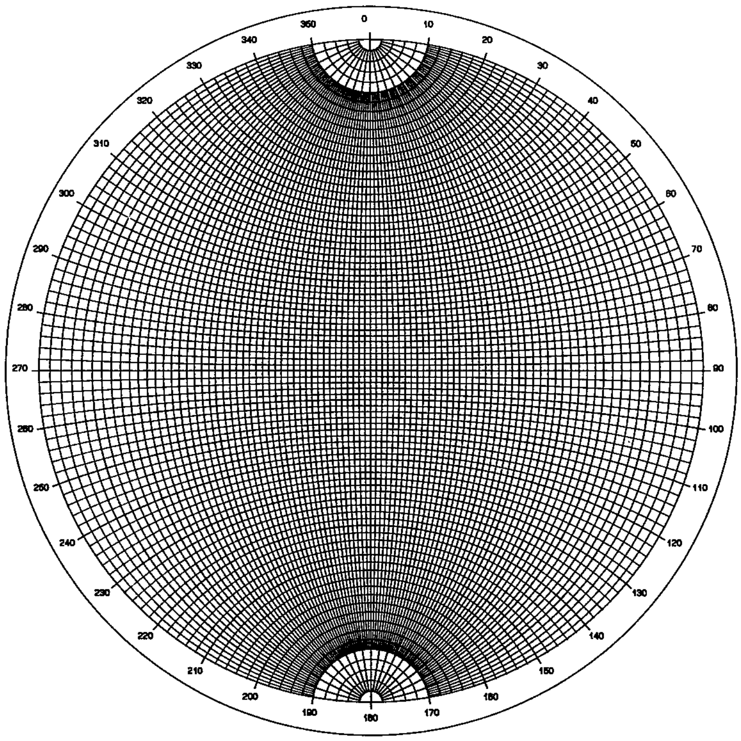
RICHARDUS, P. & ADLER, R.K. (1972) — Map Projections for geodesists, cartographers and geographers. *North-Holland Pub. Comp.*, Amsterdam, 184 pp.

Roland Digital Group — Drafting Plotter GRX-400 GRX-300 Operation Manual. 384 pp.

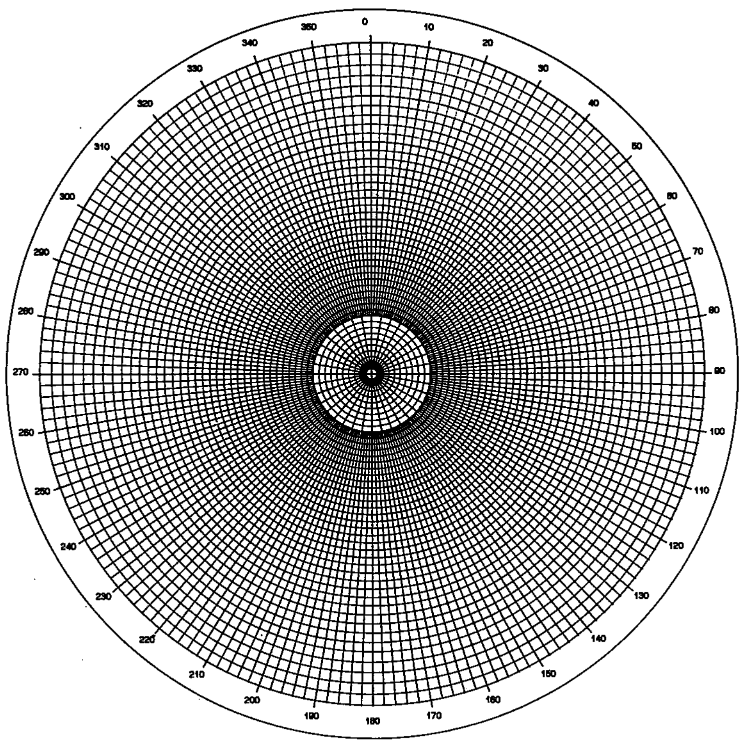
VISTELIUS, A.B. (1966) — Structural diagrams. *Pergamon Press Ltd.*, Oxford, 190 pp.

PROJECÇÃO ESTEREOGRÁFICA

REDE DE IGUAL ÂNGULO DE WULFF



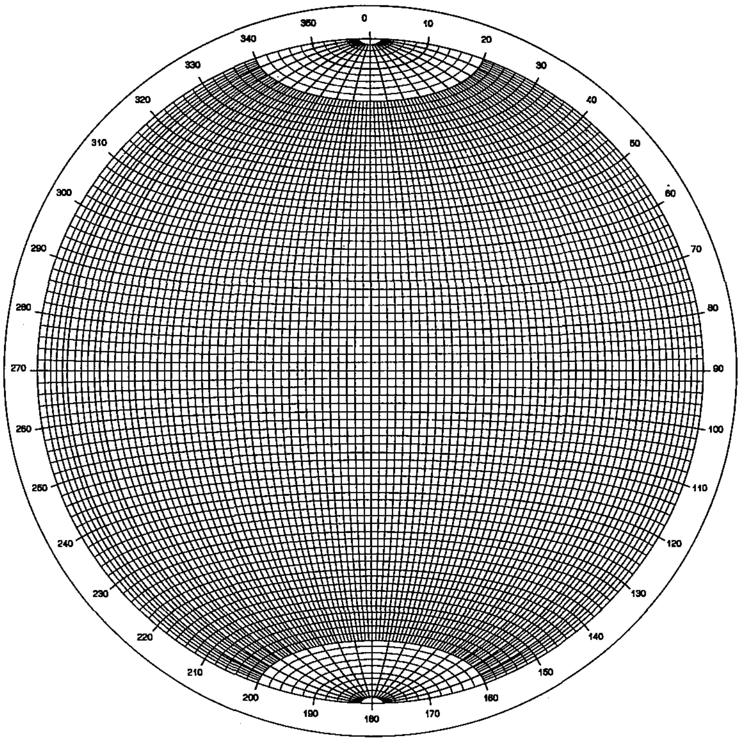
Rede equatorial



Rede polar

PROJECÇÃO DE LAMBERT

REDE DE IGUAL ÁREA DE SCHMIDT



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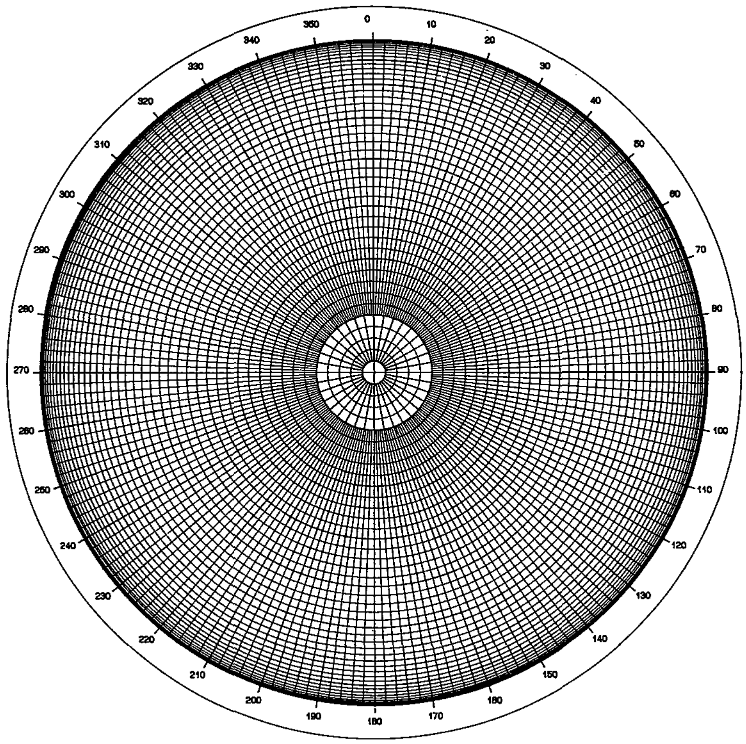
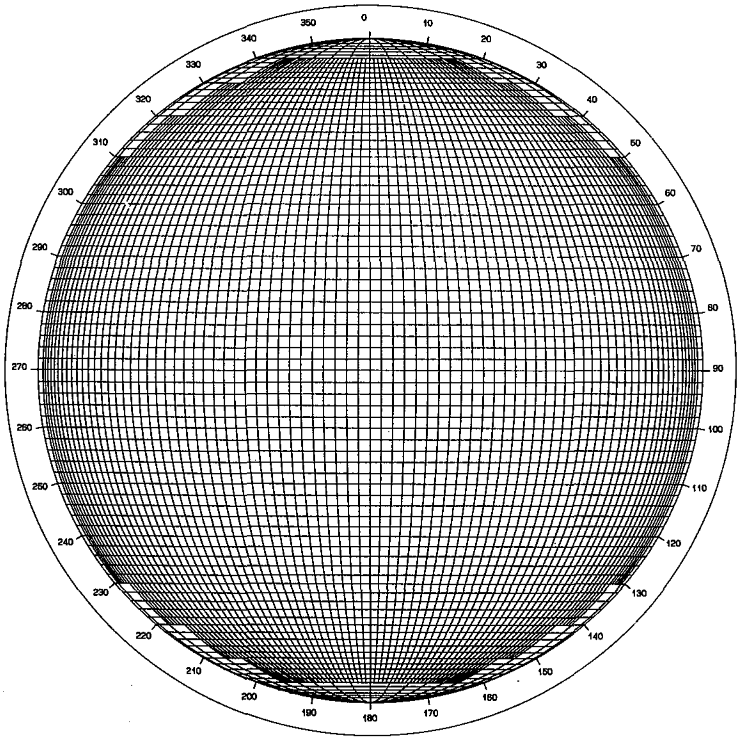
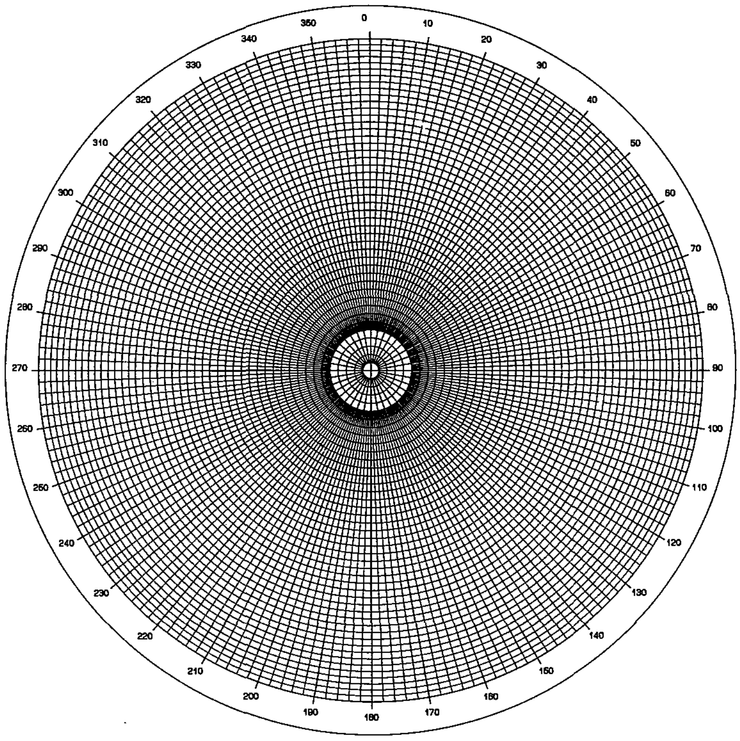


Rede polar

PROJECÇÃO ORTOGRÁFICA

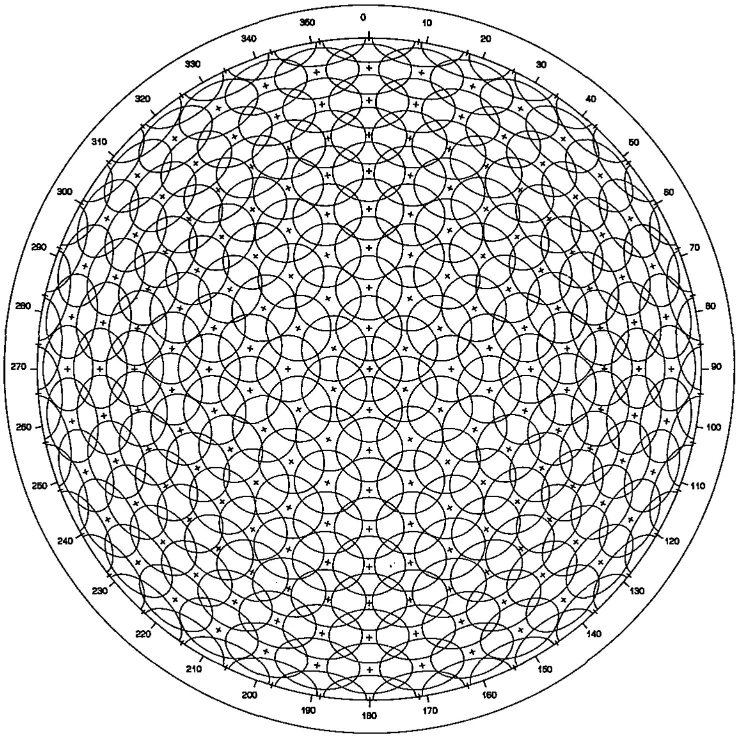


Rede equatorial



Rede polar

REDE IGAREA 220



ÁBACO ORTOGRÁFICO DTP